

Gottfried Gabriel: Frege and Others on the Justification of Basic Logical Laws

Handout

1) In the end there are axioms and postulates or, in a word, *basic* principles, which can't be proved and don't need a proof. (Leibniz: *Monadology*, § 35)

(2) Every proof is a syllogism, or a chain of syllogisms, which completes the premises required for the given proposition *T*, so that they fit into one another in such a way that *T* follows as their necessary consequence. But the validity of every conclusion depends upon the validity of its premises: these again might be established by fresh proofs, but this procedure would go on *ad infinitum* without any result were there not a number of universal propositions which we accept as immediate truths, which therefore neither need nor are capable of proof [*wenn es nicht irgend eine Anzahl allgemeiner Sätze gäbe, deren Gültigkeit für uns unmittelbar feststeht, die daher eines Beweises weder bedürftig noch fähig sind*], but are themselves the ultimate grounds by appeal to which we may decide in every case whether a conclusion is correctly or incorrectly drawn from its premises. I do not intend as yet to discuss the question of the source from which we obtain these immediate truths: we are here concerned only with the mark which justifies us in classing a proposition *T* among the *axioms*, assent to which we believe ourselves entitled to demand from every sane person. Now it is conceivable that, just because there is no possible proof [*Beweis*] of an axiom, this mark may in the last resort be nothing but the *self-evidence*, the immediate clearness and certainty with which the content of a universal proposition thrusts itself upon us as a necessity of thought; and in fact this is what we always come back to in the end. (Lotze: *Logik*, § 200)

(3) When a straight line intersects one of two parallel lines, does it always intersect the other? This question, strictly speaking, is one that each person can only answer for himself. I can only say: so long as I understand the words 'straight line', 'parallel' and 'intersect' as I do, I cannot but accept the parallels axiom. If someone else does not accept it, I can only assume that he understands these words differently. Their sense is indissolubly bound up with the axiom of parallels. (Frege: *Posthumous Writings*. Translated by Peter Long and Roger White. Oxford 1979, p. 247)

(4) I have never concealed from myself that it is not as obvious (*nicht so einleuchtend ist*) as the others nor as obvious as must properly be required of a logical law. (Frege: *Basic Laws of Arithmetic*, vol. II, p. 253)

(5) Whether a truth is an axiom depends therefore on the system, and it is possible for a truth to be an axiom in one system and not in another. [...] We can see from this that the possibility of one system does not necessarily rule out the possibility of an alternative system, and that we may have a choice between different systems. So it is really only relative to a particular system that one can speak of something as an axiom. (Frege: *Posthumous Writings*, p. 205f.)

(6) All the propositions of logic are of equal status: it is not the case that some of them are essentially primitive propositions and others essentially derived propositions. (Wittgenstein: *Tractatus* 6.127)

(7) It is clear that the number of the 'primitive propositions of logic' [der "logischen Grundgesetze"] is arbitrary, since one could derive logic from a single primitive proposition, e. g. by simply constructing the logical product of Frege's primitive propositions. (Frege would perhaps say that we should then no longer have an immediately self-evident primitive proposition. But it is remarkable that a thinker as rigorous as Frege appealed to the degree of self-evidence as the criterion of a logical proposition.) (Wittgenstein: Tractatus 6.1271)

(8) Any judgment that can be justified at all can be justified by a deductive derivation: he [Frege] does not allow for the possibility of any other form of justification. (M. Dummett: Frege. Philosophy of Mathematics. Cambridge, Mass. 1991, p. 25)

(9) As to the question, why and with what right we acknowledge a logical law to be true, logic can respond only by reducing it to other logical laws. Where this is not possible [in the case of *basic* logical laws], it can give no answer. Stepping outside logic [i. e., if we move into epistemology], one can say: our nature and external circumstances force us to judge, and when we judge we cannot discard this law – of identity, for example – but have to acknowledge it if we do not want to lead our thinking into confusion and in the end abandon judgement altogether. I neither want to dispute nor to endorse this opinion, but merely note that what we have here is not a logical conclusion [*logische Folgerung*]. What is offered here is not a ground of being true [*Grund des Wahrseins*] but of our taking to be true [*unseres Fürwahrhaltens*]. (Frege: Basic Laws of Arithmetic. Translated and edited by Philip A. Ebert and Marcus Rossberg with Crispin Wright. Oxford 2013, vol. I, p. XVII.)

(10) Now the grounds which justify the recognition of a truth often reside in other truths which have already been recognized. But if there are [*Wenn aber*] in the sense of "But since there are"] any truths recognized by us at all, this cannot be the only form that justification takes. There must be judgements whose justification rests on something else, if they stand in need of justification at all [wenn sie überhaupt einer solchen bedürfen].

And this is where epistemology comes in. Logic is concerned only with those grounds of judgement which are truths. (Frege: Posthumous Writings, p. 3.)

(11) The thought loses value for us as soon as we recognize that the *Bedeutung* of one of its parts is missing. (On Sense and *Bedeutung*, p. 33; The Frege Reader, ed. M. Beany, p. 157)

(12) Seen from 'our' point of view, the logical is an 'ought': but this ought has to have its ground in something which is valid in itself and which becomes an ought, or a rule for us, only in its relation to a consciousness which can err. (Windelband: Die Prinzipien der Logik. Tübingen 1912, p. 18.)

(13) The ambiguity [*Doppelsinn*] of the word 'law' here is fatal. In one sense it states what is, in the other it prescribes what ought to be. [...] Every law stating what is the case can be conceived as prescriptive, one should think in accordance with it, and in that sense it is accordingly a law of thought. (Frege: Basic Laws of Arithmetic, vol. I, p. XV)

(14) The validity of a proposition is the ground of our believing it to be true, and not the other way round, as if our believing something to be true would make it valid.
(Windelband: *Der Wille zur Wahrheit*. Heidelberg 1909, p. 25)

(15) And furthermore, this impossibility of our rejecting the law [of identity] does not prevent us from supposing that there are beings who do reject it; but it does prevent us from supposing that these beings are right in doing so; it also prevents us from doubting whether we or they are right. At least this goes for me. [Here again a subject comes in, G. G.] If others dare to recognize and doubt a law in the same breath, then it seems to me like trying to jump out of one's own skin, against which I can only urgently warn. Anyone who has once recognized a law of truth has thereby also recognized a law that prescribes how judgment should be made, wherever, whenever and by whomever they may be made. (Frege: *Basic Laws of Arithmetic*, vol. I, p. XVII)