THE PRIORITY OF PROPOSITIONS IS THERE A RUSSELL'S PARADOX?

María J. Frápolli

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Universidad de Granada

ANIMATING IDEAS

What is Analytic Philosophy?

Is Contemporary Logic the legit heir of Frege's work?

WE
PHILOSOPHERS

What do we do when we do philosophy?

What do we do when we do logic?

As philosophers, do we have a social responsibility? Is philosophy a game with words?

Did we get everything wrong?

The Priority of Propositions.
A Pragmatist Philosophy of Logic

The Big Question:

Who are we? What kind of animals are we?

The Right Answer:

We are the ones who say "we" (Brandom 1994, p.4)

We are agents of truth (Sokolowsky 2008, p. 1)

THE FRAMEWORK

To be *rational* is to be responsive to *reasons*. Human beings are, in essence, both *producers* and *consumers* of reasons. Reasons are *propositions*—the contents of certain mental states and linguistic acts. Alternative terms include *judgeable contents*, *what is said*, or the *lekton*. On this view, propositions have priority over concepts, which are identified and individuated within propositions.

Kant, Hegel, Frege, Wittgenstein, Grice, Brandom

THE PLAN

1. Identify some semantic principles characteristic of a pragmatist approach

Core principles that derive from the centrality of practices and the idea that meaning is rooted in inferential roles

2. Contrast with representationalism and semantic atomism

Pragmatist principles represent a paradigm shift in contrast to traditional views like representationalism (which treats meaning primarily in terms of reference) and semantic atomism (which treats concepts or words as having meaning independently of their role in larger structures).

THE PLAN

3. Apply the pragmatist framework to the Frege–Russell exchange

Examine the exchange between Frege and Russell concerning the paradox of the predicate that cannot be predicated of itself. *Interpret the paradox as a test case for competing semantic frameworks.*

4. Assess whether these semantic principles dissolve the difficulty

Investigate whether the pragmatist perspective that gives priority to assertable contents (*propositions*) can dissolve the paradox.

THE PLAN RUNNING IN THE BACKGROUND

1. Identify pragmatic principles *in Frege's work*

To uncover pragmatic elements within Frege's philosophy of language—principles that emphasize the role of *use*, *assertion*, *inferential articulation*, and the *context principle* in determining meaning.

2. Re-interpret Frege's semantics in light of these principles

Suggest an alternative reading of Frege's semantics: one that treats the sense (in a sense of "sense") of an expression as something tied to its role in reasoning and assertion.

THE PLAN RUNNING IN THE BACKGROUND

3. Examine whether Frege later rejected these principles

The next step is to determine whether Frege's "mature" work, especially in the *Grundgesetze*, systematically excludes or suppresses these earlier pragmatic insights. The question is whether Frege *abandoned* the pragmatic/*rationalist* core of his earlier semantic insights, or whether these remained silently operative even in his formal logical work.

4. Apply this framework to the Frege–Russell exchange

With this background in place, revisit the famous exchange with Russell over the paradox of the predicate of all predicates that cannot be predicated of themselves

THE PLAN RUNNING IN THE BACKGROUND

5. Assess whether Frege had the tools to answer Russell

See whether Frege's own system, once its pragmatic resources are brought to light, could have addressed Russell's challenge more effectively than is usually assumed.

6. Evaluate the scope of the paradox

Finally, determine whether the Russell paradox truly undermines Frege's entire philosophical project, or whether its destructive force is confined to specific components of his *Grundgesetze*, such as the (*imprecise, careless*) formal treatment of sets and extensions. On this view, Frege's semantics, logic, and philosophy of language might remain largely intact—even strengthened—by recovering the pragmatic dimension of his thought.

FIVE FREGEAN PRINCIPLES

THE ASSERTION PRINCIPLE (PA)

THE CONTEXT PRINCIPLE (CP)

THE PRINCIPLE OF PROPOSITIONAL PRIORITY (PPP)

THE PRINCIPLE OF GRAMMAR SUPERSEDING (PGS)

THE PRINCIPLE OF INFERENTIAL INDIVIDUATION (PII)

ASSERTION, CONTEXT AND GRAMMAR

(PA) (*The Principle of Assertion*): Assertion is the minimal act required to produce outputs with logical properties

(CP) (*The Context Principle*): Only in the context of a sentence does a word have meaning.

(PGS) (*The Principle of Grammar Superseding*): Grammatical analysis is not a source of logical knowledge. ⏩

GRAMMAR SUPERSEDING

Several relationships between sentences and their senses (one-to-one and many-to-one) ➤➤

Several senses of "sense" with different identification and individuation criteria (Bell 1979, Cresswell 2002, Dummett 1973, 1981, Penco 2003)

PROPOSITIONAL PRIORITY AND INFERENCE

(PPP) (*The Principle of Propositional Priority*): The primary bearers of logical relations are propositions and sets of propositions.

(PII) (*The Principle of Inferential Individuation*): Two sentences (uttered by an agent in context) express one and the same proposition if and only if their contents follow from the same set of propositions, and the same set of propositions follows from them.

INFERENTIAL INDIVIDUATION

All logically equivalent sentences express one and the same thought (letter to Husserl ≠ Dummett)

Thoughts as non-structured entities

RUSSELL'S PARADOX

Russell to Frege 16.6.1902 (1980, p. 130-1)

I have encountered a difficulty only on one point. You assert that a function could also constitute the indefinite element. This is what I used to believe, but this view now seems to me dubious because of the following contradiction: Let w be the predicate of being a predicate which cannot be predicated of itself. Can w be predicated of itself? From either answer follows its contradictory. **We must therefore conclude that w is not a predicate**. Likewise, **there is no class (as a whole) of those classes which, as wholes, are not members of themselves**. From this I conclude that under certain circumstances a **definable** set does not form a whole.

ABSTRACT ENTITIES AND THEIR EXPRESSION

For Frege: "propositions" (sentences, *Sätze*) are the representation of thoughts, their vehicles

For Russell: propositions are the thoughts themselves
(Russell) (28.12.1902, p. 153). Frege explains the difference in (PW 1899-1906, 1924).

Classes ≠ systems and wholes

1. The elements of a class are not elements of the the classes to which the first one belongs

2. The parts of parts of a whole are parts of the whole

ABSTRACT ENTITIES AND THEIR EXPRESSION

"The only members of the class of prime numbers are the prime numbers, but not the class of prime numbers of the form 4n+1" (Frege to Russell 28/07/1902) [∈ ≠ ⊂]

Classes, extensions (≠ meanings) of concepts, are logical objects. Their members are always determined (ibid.)

TWO STRATEGIES TO BLOCK THE PARADOX

Bottom-Up (Zoom-in) Strategy:

The difference between concept and object; thoughts and sentences need a saturated and an unsaturated part; concepts (predicates) cannot be subjects of sentences, but their names can.

Top-Down (Zoom-Out) Strategy:

The Principles of Assertion, Context and Propositional Priority: a concept must be identified in an assertable judgeable content. A combination of words is not enough.

BOTTOM-UP STRATEGY: ABSTRACT ENTITIES

Frege to Husserl (WB 1891): predicates have *Sinn, Bedeutung* and *Umfang* (Penco 2003)

1. Concept-words/predicates have concepts as **Bedeutungen**;

2. Concepts-words, as unsaturated expressions, cannot be the subject of a sentence.

3. Concepts cannot take the place of objects in **Gedanken**

BOTTOM-UP STRATEGY

Incidentally, the expression 'A predicate is predicated of itself' does not seem exact to me. A predicate is as a rule a first-level function which requires an object as argument, and which cannot therefore have itself as argument (subject). Therefore, I would rather say: **A concept is predicated of its own extension.** If the function $\boldsymbol{\Phi}(\boldsymbol{\xi})$ is a concept, I designate its extension (or the pertinent class) by '$\grave{\varepsilon}\boldsymbol{\Phi}(\boldsymbol{\varepsilon})$' (though I now have some doubts about the justification for this). '$\boldsymbol{\Phi}(\grave{\varepsilon}\boldsymbol{\Phi}(\boldsymbol{\varepsilon})$' or '$\boldsymbol{\Phi}(\grave{\varepsilon}\boldsymbol{\Phi}(\boldsymbol{\varepsilon})\bigcap \grave{\varepsilon}\boldsymbol{\Phi}\boldsymbol{\varepsilon}$)' is then the predication of the concept $\boldsymbol{\Phi}(\boldsymbol{\xi})$ of its own extension. Frege to Russell 22.6.1902 (p.132-3)

[*Step 1*]

PRONOUNS: RAMSEY TO RESCUE

"what he believed is true" ≈ "if p was what he believed, p" [*If she believes something, it**]

We claim to have defined truth we ought to be able to substitute our definition for the word 'true' wherever it occurs. But the difficulty we have mentioned renders this impossible in ordinary language which treats what should really be called pro-sentences as if they were pronouns. (Ramsey 1929/2001, p. 437)

PGS: Pronouns in natural languages and variables in formal languages hide the logical status of their contents. They convey the false impression that they stand for objects, but "p" in Ramsey's characterisation, and "w" in Russell's letter ["Let w be the predicate of being a predicate which cannot be predicated of itself"] do not stand for objects.

If "w" is a genuine proper name, Russell's sentence is false since it is not a predicate. If it is a predicate, "w is the predicate of being a predicate which cannot be predicated of itself" is not a sentence. A predicate cannot be a subject.

BOTTOM-UP STRATEGY

- *A concept is predicated of its own extension:* "The concept of all concepts that cannot be predicated of their own extension" does not mean anything

- "(…) is a concept that cannot be predicated of its own extension". This is the name of a concept. The concept named cannot form a **true well-formed** sentence, independently of the logical status of the argument that fill in the argument place.

  – *If the argument is a name (meaning an object), the whole sentence is false since no object can be a concept, and a fortiori a concept that cannot be predicated of their own extension.*

  – *If the argument is a concept, the complex is not a sentence since a sentence (a judgeable content) is formed out of a name (saturated entity/expression) and a function/concept/concept-word (unsaturated one).*

  [Step 2.1]

BOTTOM-UP STRATEGY

Frege in 28/07/1902 (p. 141)

The difficulty in the proposition 'A function never takes the place of a subject' is only an apparent one, occasioned by the inexactness of the linguistic expression. For the words 'function' and 'concept' should properly speaking be rejected. Logically, they should be names of second-level functions, but they present themselves linguistically as names of first-level functions. It is not surprising that we run into difficulties in using them. [...] If we want to express ourselves precisely, our only option is to talk about words or signs.

["(…) is a function" ⇒ "(…) +2", "(…) follows 3 in a series"]

BOTTOM-UP STRATEGY

Frege in 29/06/1902 (p. 136)

- Russell: "ξ can never take the place of a proper name" is a false ~~proposition~~ (sentence) if ξ is a proper name (*Frege grants this*), but otherwise it is not a ~~proposition~~ (sentence) at all (*Frege rejects this*).

- *The difficulty here lies in the difference between use and mention, hidden by the use of variables*

- Frege: What is correct is that *if "ξ" is not a proper name, then "ξ can never take the place of a proper name"* is not a ~~proposition~~ (sentence).

BOTTOM-UP STRATEGY

Frege in 29/06/1902 (p. 136)

Frege: A right substitution for "ξ" is "'(...).3+4'", with two pairs of quotations marks (*Frege's remark*). While "(...).3+4" is a function name, "'(...).3+4'" is a proper name and its meaning is the function name "(...).3+4".

In "Something is a concept that cannot be predicated of its own extension", "something" stands for an object, "for a function name cannot take the place of "something"".

[*Thus: "Something is a concept that can/cannot be predicated of its own extension" is always false. It is a category mistake like "Julius Caesar is prime"*]
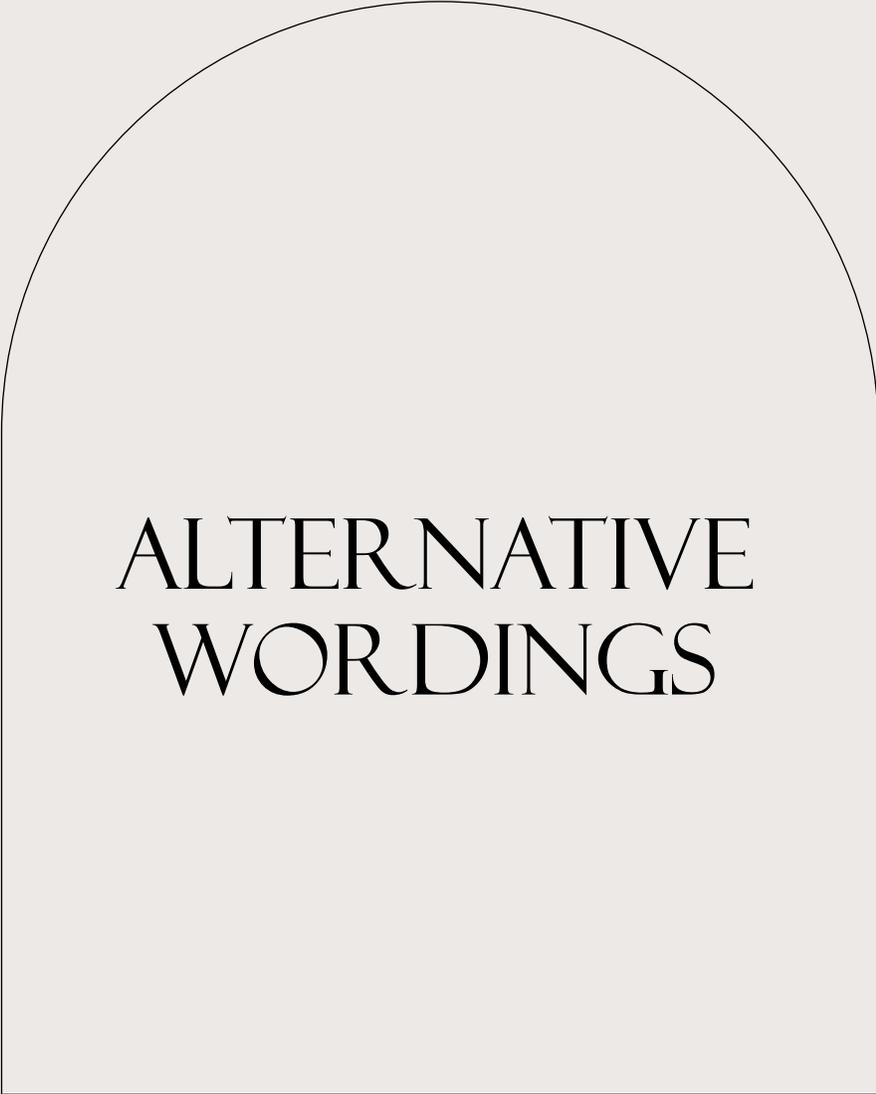
ALTERNATIVE WORDINGS

The class of all concepts that cannot be predicated of their own extension [= w]

[*Step 2.2*]:

Can "w" be **predicated** of its own extension? A class cannot be predicated of anything, a class is an object not a concept.

[*Step 2.3*]:

Can "w" be a **member** of its own extension? Concepts are the members of this extension, and a class is an object, not a concept.

ALTERNATIVE WORDINGS

[*Step 2.4*]:

The class of all classes that do not belong to themselves. Does this class belong to itself?

Frege 28/07/1902:
"[I]f we are given a whole, it is not yet determined what we are to envisage as its parts [...]. On the other hand, if we are given a class, it is determined what objects are members of it. The only members of the class of prime numbers are the prime numbers, but not the class of prime numbers of the form 4n+1, for this class is not a prime number."
"The objects that are members of a class can at the same time form a system. But the system must still be distinguished from the class. [...] It seems to me that you want to admit only systems and not classes"

TOP-DOWN STRATEGY

Frege 28/07/1902:

If we have a relation $\Phi(\xi, \zeta)$ from which the following propositions hold: (1) from $\Phi(a, b)$ we can infer $\Phi(b, c)$, and (2) from $\Phi(a, b)$ and $\Phi(b, c)$ we infer $\Phi(a, c)$, then this relation can be transformed into an equality (identity), and $\Phi(a, b)$ can be replaced by writing, e.g., "§a = §b" […]. But the difficulties here are not the same as in transforming the generality of an identity into an identity of ranges of values.
[28/07/1902]

TOP-DOWN STRATEGY

If we want to express ourselves precisely, our only option is to talk about words or signs. We can analyse the proposition '3 is a prime number' into '3' and 'is a prime number'. These parts are essentially different: the former complete in itself, the latter in need of completion. Likewise, we can analyse the proposition '4 is a square number' into '4' and 'is a square number'. Now it makes sense to fit together the complete part of the first proposition with that part of the second proposition which is in need of completion [..] but is makes no sense to fit together the two complete parts; they will not hold together; and it makes just as little sense to put 'is a square number' in place of '3' in the first proposition. [28/07/1902]

THE GENERALIZED CONTEXT PRINCIPLE

1. One can ask after reference only where signs are components of ~~propositions~~ (sentences) **expressing thoughts** (Frege 1893/2013, §97; Linnebø 2008, p. 100

2. A proper name has reference if, whenever it fills the argument places of a referential name of a first-level function with one argument, the resulting proper name has a reference, and if the name of a first-level function with one argument which results from the relevant proper name's filling the ξ-argument-places of a referential name of a first-level function with two argument places, always has a reference, and if the same also holds for the ζ-argument-places. (Frege 1893/2013, I, §29; Linnebø, loc. cit.)

THE GENERALIZED CONTEXT PRINCIPLE

"ἐ**Φ**(**ε**)" has a reference if, "whenever it fills the argument places of a referential name of a first-level function with one argument", say ""**Φ**(**ξ**)"", the resulting proper name, say "**Φ**(ἐ**Φ**(**ε**)", always has a reference.

It is the reference of "**Φ**(ἐ**Φ**(**ε**)" what determines whether "ἐ**Φ**(**ε**)" has a reference.

"**Φ**(ἐ**Φ**(**ε**)" must be a judgeable content, "— **Φ**(ἐ**Φ**(**ε**)", something suitable to be judged (asserted), "⊢ **Φ**(ἐ**Φ**(**ε**)", for its "parts" to be meaningful/contentful.

THE EXTENSIONAL STRATEGY

The Axiom Schema of Comprehension

(AC) $\forall w_{1\text{-}n} \exists B \forall x \, (x \in B \leftrightarrow \boldsymbol{\varphi} \, (x, w_{1\text{-}n}))$

The restricted version: The Axion Schema of Specification (Separation)

(AS) $\forall w_{1\text{-}n} \, \forall A \, \exists B \, \forall x \, (x \in B \leftrightarrow (x \in A \land \boldsymbol{\varphi} \, (x, w_{1\text{-}n}, A)$

THE CONCEPTUAL STRATEGY

The procedure of identifying concepts:

⊢ $\boldsymbol{\Phi}(\grave{\varepsilon}\boldsymbol{\Phi}(\boldsymbol{\varepsilon}))$ [PA]

– $\boldsymbol{\Phi}(\grave{\varepsilon}\boldsymbol{\Phi}(\boldsymbol{\varepsilon})$

"$\grave{\varepsilon}\boldsymbol{\Phi}(\boldsymbol{\varepsilon})$" is the name of a class, the class of the $\boldsymbol{\Phi}$s

""$\boldsymbol{\Phi}()$"" is the name of a function

A concept earns its status in the role it plays in a judgeable content. Alternative, as a part of a meaningful sentence in which the concept-word is used (≠ mentioned).

This is a global, top-down, organic, strategy that begins with propositions (i.e. premises and conclusions).

BACK TO THE PRINCIPLES

Concepts are *not* the sums of their marks (BG, p. 6, Gabriel 2013). They are the meanings (*Bedeutung*) of those parts of sentences (judgeable contents) that we regard as fix, i.e. that are unsaturated. Their argument-places are filled by saturated expressions regarded as replaceable.

(AS) is *ad hoc* to block the contradiction derived from (AC). The Fregean (PA), (CP), (PPP) and the separation between concept and objects are not. They are semantic principles, rooted some of them in Neo-Kantian insights, that that set up a framework in which Russell's paradox cannot be developed.

THE SCOPE OF THE PARADOX

Russell's paradox goes against Russell's theory of relations (Macbeth 2005) and the extensional, set-theoretic, strategy, but Frege's semantics has the conceptual tools to block it.

Even if Russell's logicism can be seen as a failure, Frege's logicism rests untouched and Frege's project encloses some clues to redirect the formalist derive of contemporary logic (and semantics)