THE FATE OF FREGE'S LOGICISM

M. J. Frápolli

Lógica y Significado, 04/10/202

Two ways of blocking the paradox:

A. **The Bottom-Up (Zoom-In) Strategy**:

[The texts quoted in this part A correspond to Frege, G. (1980). *Philosophical and Mathematical Correspondence*. Oxford, Basil Blackwell]

Step 1:

A concept is predicated of its own extension.

Frege to Russell 22.6.1902 (p.132-3):

Incidentally, the expression 'A predicate is predicated of itself' does not seem exact to me. A predicate is as a rule a first-level function which requires an object as argument, and which cannot therefore have itself as argument (subject). Therefore, I would rather say: *A concept is predicated of its own extension*. If the function $\boldsymbol{\Phi}(\boldsymbol{\xi})$ is a concept, I designate its extension (or the pertinent class) by '$\dot{\varepsilon}\boldsymbol{\Phi}(\boldsymbol{\varepsilon})$' (though I now have some doubts about the justification for this). '$\boldsymbol{\Phi}(\dot{\varepsilon}\boldsymbol{\Phi}(\boldsymbol{\varepsilon})$' or '$\boldsymbol{\Phi}(\dot{\varepsilon}\boldsymbol{\Phi}(\boldsymbol{\varepsilon})\cap \dot{\varepsilon}\boldsymbol{\Phi}\boldsymbol{\varepsilon}$') is then the predication of the concept $\boldsymbol{\Phi}(\boldsymbol{\xi})$ of its own extension.

Step 2:

A concept can be predicated of its own extension: an extension is an object, and concepts can be (truly or falsely) predicated of objects [*a logical object does not have parts and is stable*]

Alternative 1

"The concept of all concepts that cannot be predicated of their own extension" does not mean anything

The concept "(…) is a concept that cannot be predicated of its own extension". This is the name of a concept. The concept named cannot form a *true well-formed* sentence, independently of the logical status of the argument that fill the argument place. If the argument is a name (whose meaning is an object), the whole sentence is false since no object can be a concept, and *a fortiori* a concept that cannot be predicated of their own extension. If the argument is a concept, the complex is not a sentence since a sentence (alt. a judgeable content) is formed out of a name (saturated expression/entity) and a function/concept/concept-word (unsaturated one).

This argument is hinted at by Frege in 28/07/1902 (p. 141)

The difficulty in the proposition 'A function never takes the place of a subject' is only an apparent one, occasioned by the inexactness of the linguistic expression. For the words 'function' and 'concept' should properly speaking be rejected. Logically, they should **be names of second-level functions**, but they present themselves linguistically as names of first-level functions. It is not surprising that we run into difficulties in using them. […] If we want to express ourselves precisely, our only option is to talk about words or signs.

["() is a function" ⇒ "() +2", "the successor of 3" [() follows 3 in a series] ⇒ *() +2*, *the successor of 3*]

And in 29/06/1902 (p. 136):

"ξ can never take the place of a proper name" is a false proposition (sentence) if ξ is a proper name (*Frege grants this*), but otherwise it is not a proposition (sentence) at all (*Frege rejects this*). [The difficulty here lies in the difference between use and mention, hidden by the use of variables]

According to Frege, what is correct is that *if "ξ" is not a proper name, then "ξ can never take the place of a proper name"* is not a proposition (sentence).

Frege explains: A right substitution for "ξ" is "'().3+4'", with two pairs of quotations marks (*Frege's remark*). While "().3+4" is a function name, "'().3+4'" is a proper name and its meaning is the function name "().3+4". 29/06/1902 (p. 136)

Because "'().3+4'" is a proper name, "().3+4" can take the place of the subject in a sentence.

In "Something is a concept that cannot be predicated of its own extension", "something" stands for an object, "for a function name cannot take the place of "something"" (p. 140). 29/06/1902 (p. 136)

[The sentence is false. If "something" names an object, then it can neither possess the property that defines the concept—*being a concept that cannot be predicated of itself*—nor can it fail to possess that property.]

Alternative 2:

The class of all concepts that cannot be predicated of their own extension [= w]

Can "w" be predicated of its own extension? A class cannot be predicated of anything, a class is an object not a concept.

Alternative 3:

Can "w" be a member of its own extension? *Concepts*, not objects, are the members of this extension, and a class is an object, not a concept.

Alternative 4:

The class of all classes that do not belong to themselves. Does this class belong to itself?

Classes are not systems or wholes; classes are logical objects for which it is always determined what objects are members of them (140). Russell does not distinguish between the relation between a class and its members and a whole and its parts.

Frege 28/07/1902:

"[I]f we are given a whole, it is not yet determined what we are to envisage as its parts […]. On the other hand, if we are given a class, it is determined what objects are members of it. The only members of the class of prime numbers are the prime numbers, but not the class of prime numbers of the form 4n +1, for this class is not a prime number."

"The objects that are members of a class can at the same time form a system. But the system must still be distinguished from the class. […] It seems to me that you want to admit only systems and not classes"

[If classes are allowed to work as systems, them the paradox would affect the set theoretical foundation, not the intensional foundation of arithmetic.

B. The Top-Down (Zoom-Out) strategy:

The second way concerns the top-down strategy, which put to work the Context Principle (CP) and the Principle of Propositional Priority (PPP).

Instances and applications of the Context Principle:

1. Only in the context of a sentence does a word have meaning (*The foundations of arithmetic*, 1884)
2. If we want to express ourselves precisely, our only option is to talk about words or signs. We can analyse the proposition '3 is a prime number' into '3' and 'is a prime number'. These parts are essentially different: the former complete in itself, the latter in need of completion. Likewise, we can analyse the proposition '4 is a square number' into '4' and 'is a square number'. Now it makes sense to fit together the complete part of the first proposition with that part of the second proposition which is in need of completion [..] but is makes no sense to fit together the two complete parts; they will not hold together; and it makes just as little sense to put 'is a square number' in place of '3' in the first proposition. [*Correspondence*, 28/07/1902]
3. One can ask after reference only where signs are components of propositions expressing thoughts' (Frege, *The Basic Laws*, 1893/2013, §97; Linnebø 2008, p. 100).
4. The Generalised Context Principle: A proper name has reference if, whenever it fills the argument places of a referential name of a first-level function with one argument, the resulting proper name has a reference, and if the name of a first-level function with one argument which results from the relevant proper name's filling the ξ-argument-places of a referential name of a first-level function with two argument

> places, always has a reference, and if the same also holds for the ζ-argument-places. (Frege 1893/2013, I, §29; Linnebø, loc. cit.)

Frege 28/07/1902:

If we have a relation **Φ** (ξ, ζ) from which the following propositions hold: (1) from **Φ** (a, b) we can infer **Φ** (b, c), and (2) from **Φ** (a, b) and **Φ** (b, c) we infer **Φ** (a, c), then this relation can be transformed into an equality (identity), and **Φ** (a, b) can be replaced by writing, e.g., "§a = §b" […]. But the difficulties here are not the same as in transforming the generality of an identity into an identity of ranges of values.

The Axiom Schema of Comprehension

(AC) $\forall w_{1\text{-}n} \exists B \forall x\ (x \in B \leftrightarrow \boldsymbol{\varphi}\ (x, w_{1\text{-}n})$

The restricted version: The Axion Schema of Specification (Separation)

(AS) $\forall w_{1\text{-}n}\ \forall A\ \exists B\ \forall x\ (x \in B \leftrightarrow (x \in A \land \boldsymbol{\varphi}\ (x, w_{1\text{-}n}, A)$

The membership relation is symmetrical (homogeneous) concerning the logical status of its two arguments. Hence, "A ∈ A" is a well-formed formula. The relation between object and concept when an object falls under a concept is not. It can be represented as "Pa", or a predication, but the two components have and must have different (exclusive) logico-semantic statuses.

REFERENCES:

Bell, D. (1979). *Frege's Theory of Judgement*. Oxford University Press

Brandom, R. (1994), *Making it Explicit., Making it explicit: reasoning, representing, and discursive commitment*. Cambridge, Mass.: Harvard University Press

Cresswell, M. J. (2002). Why Propositions Have No Structure. *Noûs* 36, 643-662.

Dummett, M. (1973). *Frege's Philosophy of Language*. Duckworth, London

Dummett, M. (1987). *The Interpretation of Frege's Philosophy*. Duckworth, London

Frápolli, M. J. (2023). *The Priority of Propositions. A Pragmatist Philosophy of Language*, Springer, Synthese Library

Frege, G. (1879/1967). Begriffsschrift, a formula language, modeled upon that of arithmetic, for pure thought. In Jean van Heijenoort (1967). *From Frege to Gödel. A Source Book in Mathematical Logic, 1879-1931*. Harvard University Press, 1- 82.

Frege, G. (1893/2013). *Gottlob Frege: Basic Laws of Arithmetic.* Edited by Philip A. Ebert and Marcus Rossberg. Oxford University Press

Frege, G. (1980). *Philosophical and Mathematical Correspondence*. Oxford, Basil Blackwell

Gabriel, G. (2013), Frege and the German Background of Analytic Philosophy. In M. Beaney (ed.). *The Oxford Handbook of the History of Analytic Philosophy*. Oxford University Press

Linnebø, Ø (2008). Compositionality and Frege's Context Principle. Ms., invited talk given at the CSMN Workshop on Reference (Oslo 2007) and London Logic and Metaphysics Forum (November 2007)

Linnebø, Ø (2019). The Context Principle in Frege's *Grundgesetze*. In Ebert, P. and Rossberg, M. (2019), 90-114

Macbeth, D. (2005). *Frege's Logic*. Cambridge, Mass. and London, England, Harvard University Press (kindle edition).

Penco, C. (2003). Frege: Two Theses, Two Senses. *History and Philosophy of Logic* 24 (2), 87-109

Sokolowski, R. (2008), *Phenomenology of the Human Person*. Cambridge University Press